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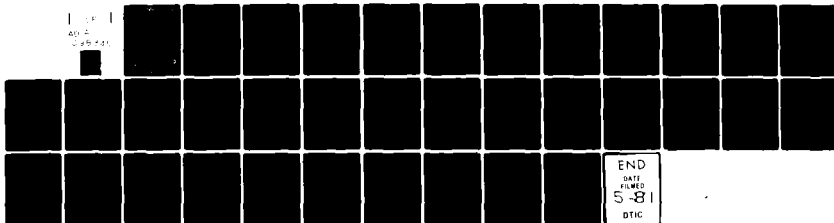
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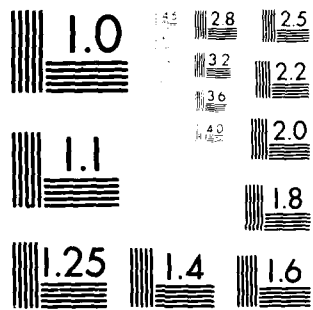
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THE TRIPOLE ANTENNA:
AN ADAPTIVE ARRAY WITH
FULL POLARIZATION FLEXIBILITY

R.T. Compton, Jr.

The Ohio State University
ElectroScience Laboratory

Department of Electrical Engineering
Columbus, Ohio 43212

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The performance of an adaptive array using three mutually perpendicular dipoles (a "tripole") is studied. A desired signal and an interference signal, each with arbitrary angle of arrival and polarization, are assumed incident on the array. Uncorrelated thermal noise is also assumed present on each element signal. The output desired signal-to-interference-plus-noise ratio (SNIR) is computed as a function of the signal arrival angles and polarizations. It is shown that, for most angles of arrival and polarizations, the array has an excellent ability to protect a desired signal from interference. Certain special cases		

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where the performance is not good are discussed in detail.

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I. INTRODUCTION

Adaptive arrays with cross-polarized elements can adapt not only to the angle of arrival of signals, but also to their polarizations. In a previous paper¹, the author described the performance of an adaptive array consisting of two pairs of crossed dipoles separated a half wavelength. It was shown that such an array can protect a desired signal from interference signals from almost all directions and with almost all polarizations. Performance was shown to be poor only when both the desired and interference signals arrive from the same direction and have the same polarization, or in certain other special cases with linearly polarized signals.

In this report we examine the performance of an even simpler adaptive array -- one consisting of three mutually perpendicular dipoles all centered at the same location. For obvious reasons, we will refer to this three-element system as a "tripole" antenna. Such an array is extremely interesting for several reasons. First, it discriminates between signals on the basis of polarization alone. (With all elements centered at the same location, there is no interelement phase shift due to angle of arrival, as in a conventional array.) Second, we will show that such an array has a remarkable ability to protect a desired signal from interference. Finally, such an array could itself be used as a building block in larger arrays that adapt to polarization as well as angle of arrival.

In Section II of the report, we define the array geometry, characterize the desired and interference signals, and develop equations for the output SINR from the array. In Section III we present results and describe the performance of such a system. Section IV contains the conclusions.

II. FORMULATION OF THE PROBLEM

Consider an adaptive array consisting of three mutually perpendicular dipoles, all centered at the same location, as shown in Figure 1. The signal from each dipole is to be processed separately in the array. Let $\hat{x}_1(t)$, $\hat{x}_2(t)$ and $\hat{x}_3(t)$ be the complex (analytic) signals received from the x-, y- and z-oriented dipoles, respectively. In the adaptive processor, each signal $\hat{x}_j(t)$ is multiplied by a complex weight w_j and summed to produce the array output. When the weights w_j are controlled by an LMS processor^{2,3}, the steady-state weight vector, $w = (w_1, w_2, w_3)^T$, is given by

$$w = \Phi^{-1} S \quad (1)$$

where Φ is the covariance matrix,

$$\Phi = E\{X^* X^T\}, \quad (2)$$

and S is the reference correlation vector,

$$S = E\{X^* \hat{r}(t)\}. \quad (3)$$

In these equations, X is the signal vector,

$$X = (\hat{x}_1(t), \hat{x}_2(t), \hat{x}_3(t))^T, \quad (4)$$

$\hat{r}(t)$ is the complex reference signal used in the adaptive array feedback^{2,3},

T denotes transpose, "*" complex conjugate, and $E(\cdot)$ expectation.

Assume two CW signals are incident on the array, one desired and the other interference. Let θ and ϕ denote standard polar angles, as shown in Figure 1. We assume the desired signal arrives from angular direction (θ_d, ϕ_d) and the interference from (θ_i, ϕ_i) . Furthermore, each signal is assumed to have an arbitrary electromagnetic polarization*. To characterize the polarization of each signal, we make the following definitions.

*I.e. we assume each signal to be completely polarized.¹⁰ We do not consider partially or randomly polarized signals.

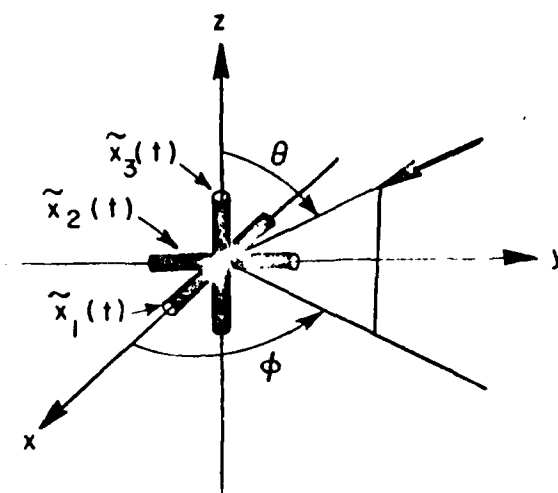


Figure 1. The tripole antenna.

Given a TEM wave propagating into the array, we consider the polarization ellipse produced by the transverse electric field as we view the incoming wave from the coordinate origin. Note that unit vectors $\hat{\phi}$, $\hat{\theta}$, $-\hat{r}$, in that order, form a right-handed coordinate system for an incoming wave. Suppose the electric field has transverse components

$$\vec{E} = E_{\phi} \hat{\phi} + E_{\theta} \hat{\theta}. \quad (5)$$

(We will call E_{ϕ} the horizontal component and E_{θ} the vertical component of the field.) In general, as time progresses, E_{ϕ} and E_{θ} will describe a polarization ellipse as shown in Figure 2. Given this ellipse, we define β to be the orientation angle of the major axis of the ellipse with respect to E_{ϕ} , as shown in Figure 2. To eliminate ambiguities, we define β to be in the range $0 \leq \beta < \pi$. We also define the ellipticity angle α to have a magnitude given by

$$|\alpha| = \tan^{-1} r \quad (6)$$

where r is the axial ratio:

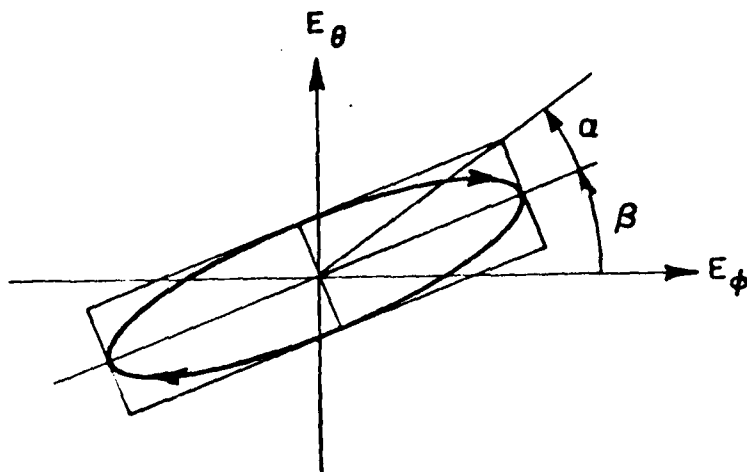


Figure 2. The polarization ellipse.

$$r = \frac{\text{minor axis}}{\text{major axis}} . \quad (7)$$

In addition, α is defined positive when the electric vector rotates clockwise and negative when it rotates counterclockwise (when the incoming wave is viewed from the coordinate origin, as in Figure 2). α is always in the range $-\frac{\pi}{4} \leq \alpha \leq \frac{\pi}{4}$. Figure 2 depicts a situation in which α is positive.

For a given state of polarization, specified by α and β , the electric field components are given by (aside from a common phase factor)

$$E_{\phi} = A \cos \gamma \quad (8a)$$

$$E_{\theta} = A \sin \gamma e^{j\eta} \quad (8b)$$

where A is the amplitude and γ and η are related to α and β by⁴

$$\cos 2\gamma = \cos 2\alpha \cos 2\beta \quad (9a)$$

$$\tan \eta = \tan 2\alpha \csc 2\beta . \quad (9b)$$

The relationship between the four angular variables α , β , γ and η is most easily visualized by making use of the Poincare Sphere⁴. This technique represents the state of polarization by a point on a sphere, such as point M in Figure 3. For a given M, 2γ , 2β and 2α form the sides of a right spherical triangle, as shown. 2γ is the side of the triangle between M and a point labeled H in the figure; H is the point representing horizontal linear polarization. Side 2β extends along the equator, and side 2α is vertical, i.e., perpendicular to side 2β . The angle η in Equations (8) and (9) is the angle between sides 2γ and 2β .^{*} The special case when $\alpha = 0$ in Equation (6) and Figure 2 corresponds to linear polarization; in this case the point M lies on the equator. If, in addition, $\beta = 0$, only E_ϕ is nonzero and the wave is horizontally polarized. This case defines the point H in Figure 3. If, instead, $\beta = \frac{\pi}{2}$, only E_θ is nonzero and the wave is vertically polarized. Point M then lies on the equator diametrically behind H. The poles of the sphere correspond to circular polarization ($\alpha = \pm 45^\circ$), with clockwise circular polarization ($\alpha = +45^\circ$) at the upper pole.

Thus, an arbitrary plane wave coming into the array may be characterized by four angular parameters and an amplitude. For example, the desired signal will be characterized by its arrival angles (θ_d, ϕ_d) , its polarization ellipticity angle α_d and orientation angle β_d , and its amplitude A_d . (I.e., A_d is the value of A in Equation (8) for the desired signal.) We will say the desired signal is defined by $(\theta_d, \phi_d, \alpha_d, \beta_d, A_d)$. Similarly, the interference is defined by $(\theta_i, \phi_i, \alpha_i, \beta_i, A_i)$.

We assume each dipole in the array is a short dipole. I.e., the output voltage from each dipole is proportional to the electric field component along the dipole. Therefore, $\hat{x}_1(t)$, $\hat{x}_2(t)$ and $\hat{x}_3(t)$ will be proportional

^{*}These relationships are derived in Reference 4. Our definitions and notation correspond exactly to those in Reference 4 if we substitute $E_\phi \rightarrow X$, $E_\theta \rightarrow Y$, $\eta \rightarrow \phi$.

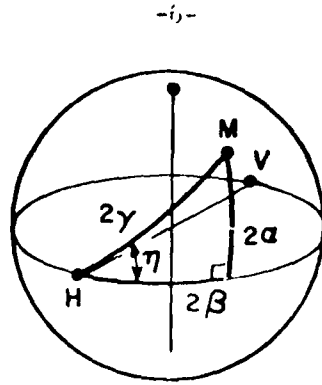


Figure 3. The Poincare sphere.

to the x-, y- and z-components, respectively, of the electric field. An incoming signal, with arbitrary electric field components E_ϕ and E_θ , has x,y,z components:

$$\begin{aligned}\vec{E} &= E_\phi \hat{\phi} + E_\theta \hat{\theta} \\ &= (E_\theta \cos\theta \cos\phi - E_\phi \sin\phi) \hat{x} + (E_\theta \cos\theta \sin\phi + E_\phi \cos\phi) \hat{y} \\ &\quad - (E_\theta \sin\theta) \hat{z} .\end{aligned}\tag{10}$$

When E_ϕ and E_θ are expressed in terms of A , γ and η as in Equation (8), the electric field components become

$$\begin{aligned}\vec{E} &= A \left[(\sin\gamma \cos\theta \cos\phi e^{j\eta} - \cos\gamma \sin\phi) \hat{x} \right. \\ &\quad \left. + (\sin\gamma \cos\theta \sin\phi e^{j\eta} + \cos\gamma \cos\phi) \hat{y} \right. \\ &\quad \left. - (\sin\gamma \sin\theta e^{j\eta}) \hat{z} \right] .\end{aligned}\tag{11}$$

Including the time dependence, we find that an incoming signal characterized by $(\theta, \phi, \alpha, \beta, A)$ produces a signal vector in the array (Equation (4)) as follows:

$$x = A e^{j(\omega t + \psi)} U, \tag{12a}$$

where U is the vector

$$U = \begin{pmatrix} \sin\gamma \cos\theta \cos\phi e^{j\eta} - \cos\gamma \sin\phi \\ \sin\gamma \cos\theta \sin\phi e^{j\eta} + \cos\gamma \cos\phi \\ -\sin\gamma \sin\theta e^{j\eta} \end{pmatrix}, \quad (12b)$$

ω is the frequency of the signal, and ψ is the carrier phase of the signal at the coordinate origin at $t=0$.

As stated above, we assume a desired signal specified by $(\theta_d, \phi_d, \alpha_d, \beta_d, A_d)$ and an interference signal specified by $(\theta_i, \phi_i, \alpha_i, \beta_i, A_i)$ are incident on the array. In addition we assume a thermal noise voltage $\hat{n}_j(t)$ is present on each signal $\hat{x}_j(t)$. The $\hat{n}_j(t)$ are assumed to be zero mean, to be statistically independent of each other, and to have power σ^2 :

$$E \left\{ \hat{n}_i^*(t) \hat{n}_j(t) \right\} = \sigma^2 \delta_{ij}, \quad (13)$$

where δ_{ij} is the Kronecker delta.

Under these assumptions, the total signal vector is given by

$$\begin{aligned} X &= X_d + X_i + X_n \\ &= A_d e^{j(\omega t + \psi_d)} U_d + A_i e^{j(\omega t + \psi_i)} U_i + X_n, \end{aligned} \quad (14)$$

where U_d and U_i are given by Equation (12b) with appropriate subscripts d or i added to each angular quantity. ψ_d and ψ_i are assumed to be random phase angles, each uniformly distributed on $(0, 2\pi)$ and statistically independent of the other. X_n is the noise vector,

$$X_n = (\hat{n}_1(t), \hat{n}_2(t), \hat{n}_3(t))^T. \quad (15)$$

The covariance matrix in Equation (2) is then given by

$$\Phi = \Phi_d + \Phi_i + \Phi_n \quad (16a)$$

where

$$\Phi_d = E\{X_d^* X_d^T\} = A_d^2 U_d^* U_d^T \quad (16b)$$

$$\Phi_i = E\{X_i^* X_i^T\} = A_i^2 U_i^* U_i^T \quad (16c)$$

and

$$\Phi_n = \sigma^2 I \quad (16d)$$

with I the identity matrix.

To make the LMS array track the desired signal, the reference signal $\tilde{r}(t)$ must be a signal correlated with the desired signal and uncorrelated with the interference^{5,6}. We assume

$$\tilde{r}(t) = A_r e^{j(\omega t + \psi_d)} \quad (17)$$

Equation (3) then yields for the reference correlation vector,

$$S = A_r A_d U_d^* \quad (18)$$

The steady-state weight vector can now be found by substituting Equations (16) and (18) into Equation (1).*

*Alternatively, one can assume that a steering vector w_0 is used in the adaptive array feedback, as described by Applebaum⁷. In that case one would choose w_0 to be the same as S in Equation (18), and the array weights are given by $w = \Phi^{-1} w_0$.

The signal-to-interference-plus-noise ratio (SINR) at the array output is then given by

$$\text{SINR} = \frac{P_d}{P_i + P_n} \quad (19)$$

where P_d is the output desired signal power,

$$P_d = \frac{1}{2} E \left\{ \left| X_d^T w \right|^2 \right\} = \frac{A_d^2}{2} \left| U_d^T w \right|^2, \quad (20)$$

P_i is the output interference power,

$$P_i = \frac{1}{2} E \left\{ \left| X_i^T w \right|^2 \right\} = \frac{A_i^2}{2} \left| U_i^T w \right|^2, \quad (21)$$

and P_n is the output thermal noise power,

$$P_n = \frac{\sigma^2}{2} |w|^2. \quad (22)$$

By making use of a matrix inversion lemma, the expression for SINR in Equation (19) can be put in the simple form:

$$\text{SINR} = \xi_d \left[U_d^T U_d^* - \frac{|U_d^T U_i^*|^2}{\xi_i + U_i^T U_i^*} \right] \quad (23)$$

where

$$\xi_d = \frac{A_d^2}{\sigma^2} = \text{desired signal-to-noise ratio (SNR)} \quad (24a)^*$$

$$\xi_i = \frac{A_i^2}{\sigma^2} = \text{interference-to-noise ratio (INR)} \quad (24b)^*$$

The derivation of Equation (23) from Equation (1) is carried out in the Appendix of Reference 11. For the particular antenna under study here, it is easily shown from Equation (12b) that

$$U_d^T U_d^* = U_i^T U_i^* = 1. \quad (25)$$

Hence in this case Equation (23) simplifies to

$$\text{SINR} = \xi_d \left[1 - \frac{|U_d^T U_i^*|^2}{\xi_i + 1} \right]. \quad (26)$$

Calculation of the SINR from Equation (26) is much easier than from Equations (19)-(22), because Equation (26) does not require calculation of the weight vector. In the next section, we show typical curves of the array performance based on Equation (26).

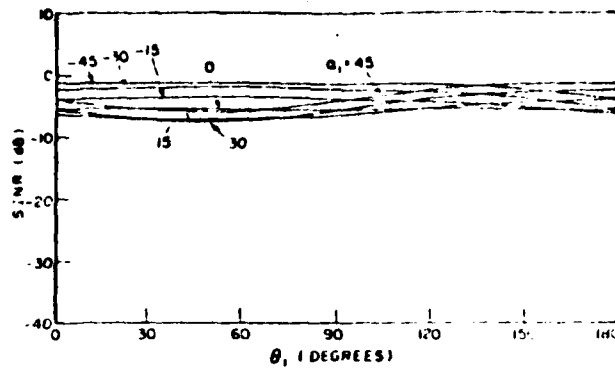
* ξ_d and ξ_i are the signal-to-noise ratios that will exist in a given array element if the incoming signal arrives broadside to that element and is linearly polarized in the direction of that element. For example, if $\alpha_d=0^\circ$ and $\beta_d=0^\circ$, the desired signal is polarized entirely in the E_ϕ -direction. Then if the signal arrives from $\phi_d=90^\circ$, the SNR on element 1 will be ξ_d . (In this case, the SNR on elements 2 and 3 will be zero.) In general, with an arbitrary state of polarization ($\alpha_d \neq 0^\circ$ or $\beta_d \neq 0^\circ$) and an arbitrary arrival angle θ_d, ϕ_d , the SNR on every element will be less than ξ_d . However, if the signals from all elements are combined with optimal weights (i.e., maximal-ratio combiner weights⁸), the total output SNR from all elements combined will be ξ_d . ξ_d is thus the maximum available SNR out of all three dipoles.

III. RESULTS

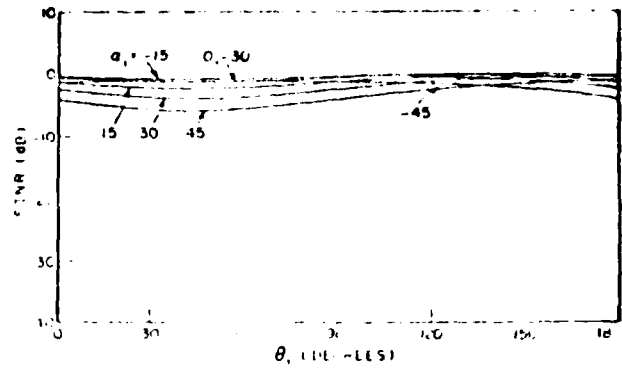
Because of the large number of parameters required to specify both the desired and interference signals, many types of curves can be plotted. We shall not present an exhaustive set of curves here. Rather, we shall first show a number of typical curves, and then will discuss the situations in which the array performance is poor.

First, we show curves representing typical performance for an arbitrarily polarized desired signal arriving from an arbitrary direction. We assume $\theta_d = \phi_d = 45^\circ$, $\alpha_d = 15^\circ$, $\beta_d = 30^\circ$ and $\text{SNR} = 0$ dB. Also, we assume $\text{INR} = 40$ dB. Figures 4 and 5 show the array output SINR as a function of the interference arrival angle for various interference polarizations. Figure 4 shows SINR versus θ_i , for $\phi_i = 45^\circ$, and Figure 5 shows SINR versus ϕ_i , for $\theta_i = 45^\circ$. Figures 4a and 5a show $\beta_i = 0^\circ$, Figures 4b and 5b show $\beta_i = 30^\circ$, and so forth, up to Figures 4f and 5f for $\beta_i = 150^\circ$. Each figure shows the SINR for $\alpha_i = -45^\circ$, -30° , -15° , 0° , 15° , 30° and 45° .

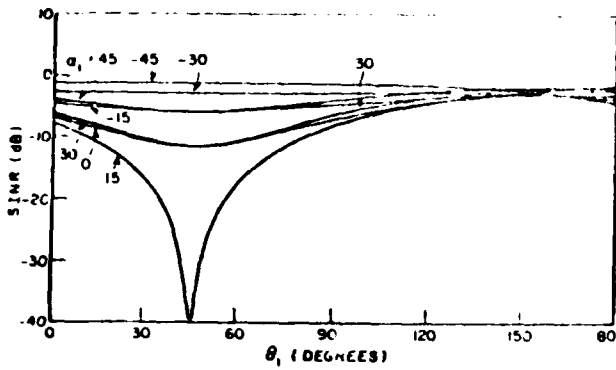
Study of these curves shows that this simple antenna system, which responds only to polarization, has a remarkable ability to protect a desired signal from interference. To cause a poor SINR, an interference signal must not only arrive from the same direction as the desired signal, it must also have the same polarization. (The relationship between SINR and polarization when both signals arrive from the same direction is discussed below under Special Case 1.) Figure 4 shows that for all θ_i not near θ_d the output SINR is above -8 dB for all α_i, β_i , and Figure 5 shows that for all ϕ_i not near ϕ_d the output SINR is above -12 dB for all α_i, β_i . Thus, with interference from these arrival angles, the array provides at least 28 dB of protection in all cases except when the conditions $\theta_i \approx \theta_d$, $\phi_i \approx \phi_d$, $\alpha_i \approx \alpha_d$ and $\beta_i \approx \beta_d$ are all simultaneously fulfilled.



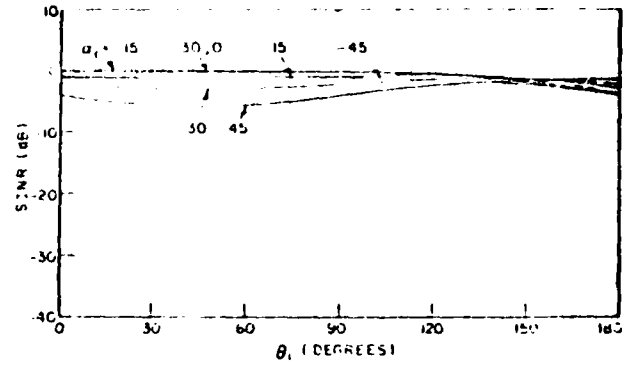
(a) $\beta_i = 0^\circ$



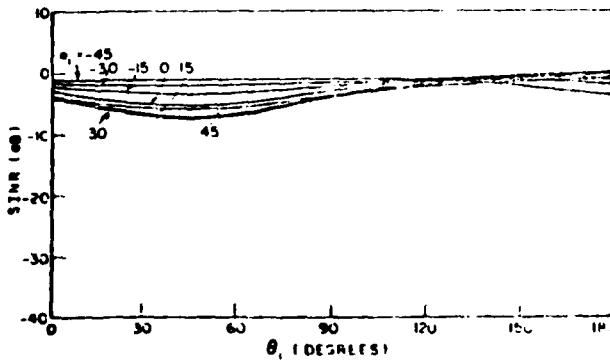
(d) $\beta_i = 90^\circ$



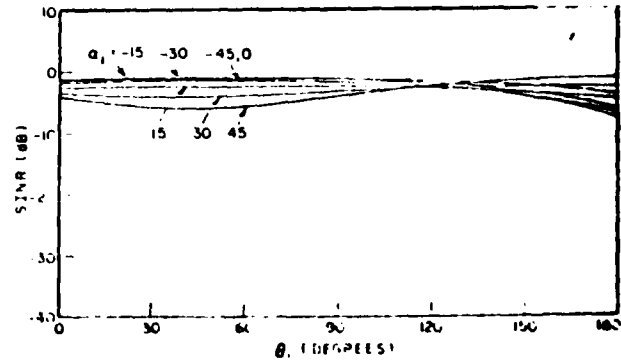
(b) $\beta_i = 30^\circ$



(e) $\beta_i = 120^\circ$

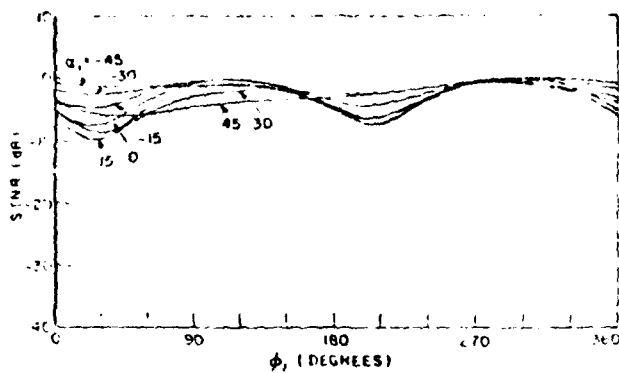


(c) $\beta_i = 60^\circ$

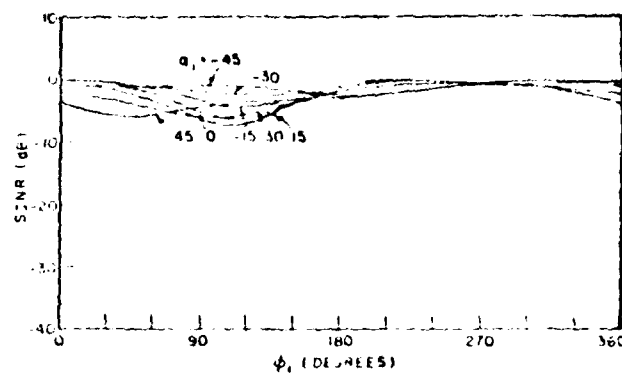


(f) $\beta_i = 150^\circ$

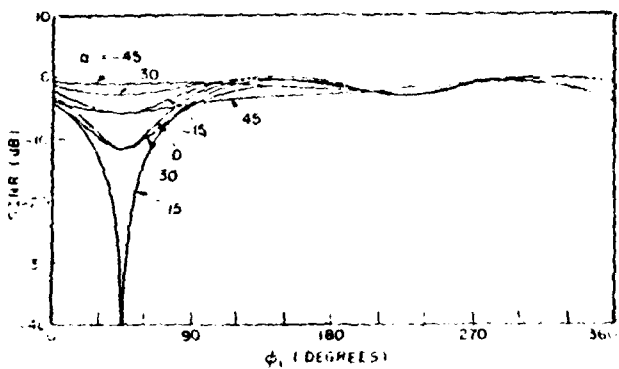
Figure 4. SINR vs. θ_i .
 $\theta_d = 45^\circ$, $\phi_d = 45^\circ$, $\alpha_d = 15^\circ$, $\beta_d = 30^\circ$, SNR = 0 dB
 $\phi_i = 45^\circ$, INR = 40 dB.



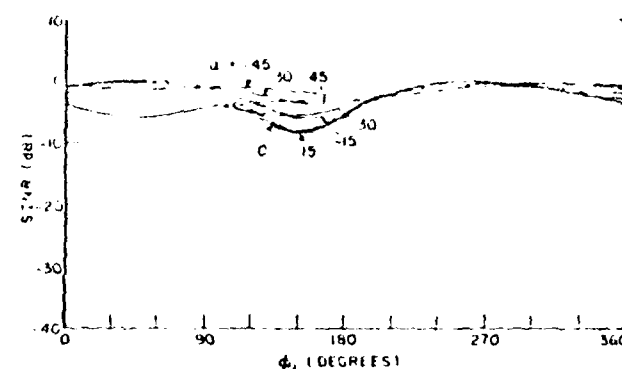
(a) $\beta_i = 0^\circ$



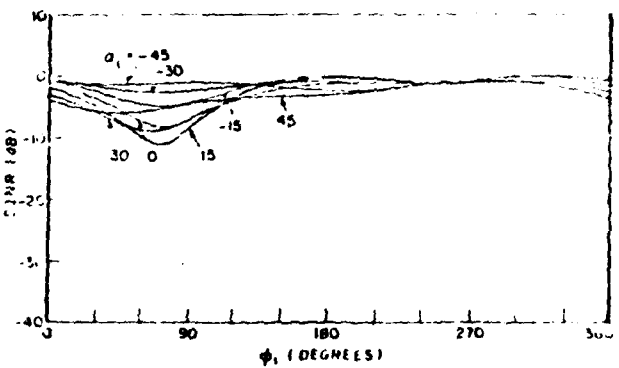
(d) $\beta_i = 90^\circ$



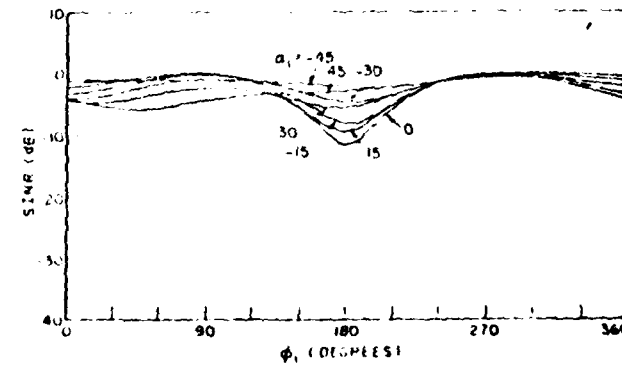
(b) $\beta_i = 30^\circ$



(e) $\beta_i = 120^\circ$



(c) $\beta_i = 60^\circ$



(f) $\beta_i = 150^\circ$

Figure 5. SINR vs. ϕ_i .

$\theta_d = 45^\circ$, $\phi_d = 45^\circ$, $\alpha_d = 15^\circ$, $R_d = 30^\circ$, SNR = 0 dB.

$\theta_i = 45^\circ$, INR = 40 dB.

When the SINR is computed for this desired signal and for other values of θ_i , ϕ_i , the results are generally similar to those shown in Figures 4 and 5. However, there is one exception. When the interference arrives from the opposite direction from the desired signal, an SINR of -40 dB can result for one particular polarization (which we call the "conjugate" polarization). This case is examined in detail below under Special Case 2.

When the performance of this array is examined for other desired signal arrival angles and polarizations, the results are again generally similar to those in Figures 4 and 5. There is, however, an important situation where the performance is not good. If the desired signal is linearly polarized, the array is vulnerable to linearly polarized interference from a wide range of angles. This situation will be examined in detail below under Special Case 3.

Thus, there are three situations where this array does not protect well against interference: (1) when both signals arrive from the same direction with the same polarization, (2) when the signals arrive from opposite directions with "conjugate" polarizations, and (3) when the desired signal has linear polarization. Let us consider these cases in detail.

Special Case 1: Both Signals Arrive From the Same Direction

When both signals arrive from the same direction, it turns out the output SINR is simply related to the separation of the two signal polarizations on the Poincare sphere. Specifically, if

$$\begin{aligned}\theta_d &= \theta_i \\ \phi_d &= \phi_i\end{aligned}\tag{27}$$

then Equation (12b) yields

$$U_d^T U_i^* = \sin \gamma_d \sin \gamma_i e^{j(\eta_d - \eta_i)} + \cos \gamma_d \cos \gamma_i \quad . \quad (28)$$

From this, using trigonometric identities, one finds that

$$\left| U_d^T U_i^* \right|^2 = \frac{1}{2} \left[1 + \cos 2\gamma_d \cos 2\gamma_i + \sin 2\gamma_d \sin 2\gamma_i \cos(\eta_d - \eta_i) \right] \quad . \quad (29)$$

Suppose M_d and M_i are points on the Poincare sphere representing the polarizations of the desired and interference signals, respectively. Then in Equation (29), $2\gamma_d$ and $2\gamma_i$ are sides of a spherical triangle with arc $M_d M_i$ as the third side, as shown in Figure 6. The angle $\eta_d - \eta_i$ is the angle opposite side $M_d M_i$. Using a well-known identity from spherical trigonometry⁹, we have

$$\cos 2\gamma_d \cos 2\gamma_i + \sin 2\gamma_d \sin 2\gamma_i \cos(\eta_d - \eta_i) = \cos(M_d M_i) \quad (30)$$

so Equation (29) is equivalent to

$$\left| U_d^T U_i^* \right|^2 = \frac{1}{2} \left[1 + \cos(M_d M_i) \right] = \cos^2 \left(\frac{M_d M_i}{2} \right) \quad . \quad (31)$$

Then from Equation (26), we have

$$\text{SINR} = \xi_d \left[\frac{1 + \xi_i \sin^2 \left(\frac{M_d M_i}{2} \right)}{1 + \xi_i} \right] \quad (32)$$

This result shows that when both signals arrive from the same angle, the SINR obtained depends only on the separation $M_d M_i$ on the Poincare sphere. The specific polarizations do not matter, only the separation.

Figure 7 shows a plot of SINR versus the spherical distance $M_d M_i$, in angular measure, for $\xi_d = 0$ dB and $\xi_i = 40$ dB. We see that a separation of $M_d M_i = 37^\circ$ is required to have $\text{SINR} = -10$ dB, i.e., for 30 dB of

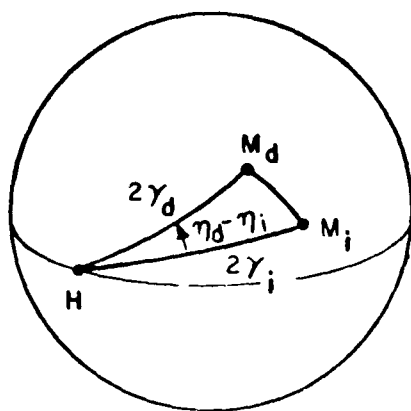


Figure 6. The points M_d and M_i .

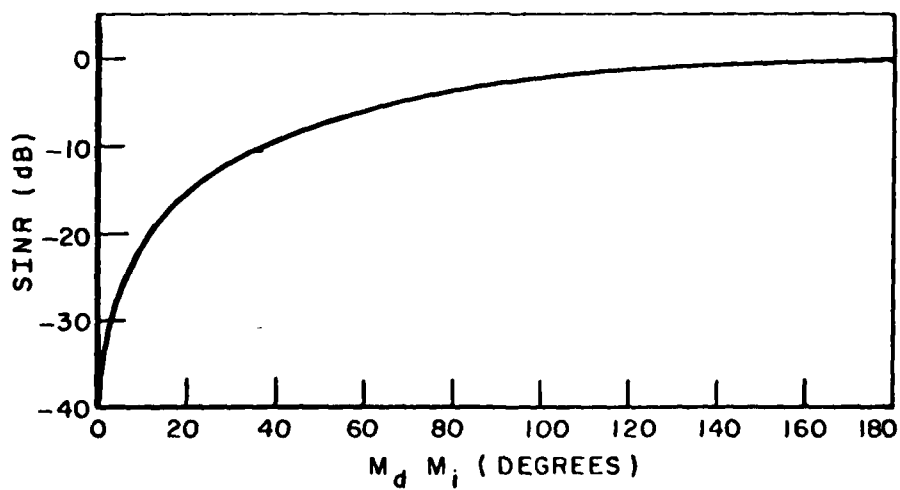


Figure 7. SINR vs. $M_d M_i$ for $\theta_i = \theta_d$, $\phi_i = \phi_d$.
(SNR=0 dB, INR=40 dB)

interference protection. Thus, for example, if $\beta_i = \beta_d$, we need $\alpha_d - \alpha_i = 18.5^\circ$ for 30 dB of protection. (Recall that point M is above the equator by 2α in Figure 3.)

Special Case 2: Signals Arrive From Opposite Directions

A poor SINR will also occur if the interference arrives from the opposite direction from the desired signal and if the interference polarization is conjugate to that of the desired signal. (We define "conjugate" below.)

First let us illustrate this situation. Figure 8 shows a calculation of SINR similar to those in Figure 5. The desired signal is again at $\theta_d = \phi_d = 45^\circ$ with $\alpha_d = 15^\circ$, $\beta_d = 30^\circ$, SNR = 0 dB and INR = 40 dB. Figure 8 shows SINR versus ϕ_i , but now for $\theta_i = 135^\circ$ instead of 45° as in Figure 5, and for $\beta_i = 150^\circ$. It may be seen that the SINR drops to -40 dB when $\phi_i = 225^\circ$ and $\alpha_i = -15^\circ$. Note that the angle of arrival $\theta_i = 135^\circ$, $\phi_i = 225^\circ$ is exactly in the opposite direction from the desired signal, with $\theta_i = \phi_d = 45^\circ$. Moreover, as will be explained below, the polarization $\alpha_i = -15^\circ$, $\beta_i = 150^\circ$ is conjugate to that of the desired signal with $\alpha_d = 15^\circ$ and $\beta_d = 30^\circ$.

Consider the general case. When the interference arrives from the opposite direction to the desired signal, we have

$$\theta_i = 180^\circ - \theta_d$$

and

$$\phi_i = \phi_d + 180^\circ \quad (33)$$

For these values of θ_i and ϕ_i , one finds from Equation (12b) that

$$U_d^T U_i^* = \sin \gamma_d \sin \gamma_i e^{j(\eta_d - \eta_i)} - \cos \gamma_d \cos \gamma_i \quad (34)$$

This differs from Equation (28), when both signals arrive from the same direction, only in the minus sign preceeding $\cos \gamma_d \cos \gamma_i$.

Given a desired signal polarization γ_d, η_d , let us define a new polarization with parameters γ_d^*, η_d^* given by

$$\gamma_d^* = \gamma_d,$$

and

$$\eta_d^* = \eta_d \pm 180^\circ, \quad (35)$$

where we choose the sign to keep η_d^* in the range $-180^\circ \leq \eta_d^* \leq 180^\circ$. In terms of γ_d^* and η_d^* , $U_d^T U_i^*$ in Equation (34) may be written

$$U_d^T U_i^* = -\sin \gamma_d^* \sin \gamma_i e^{j(\eta_d^* - \eta_i)} - \cos \gamma_d^* \cos \gamma_i \quad (36)$$

Therefore $|U_d^T U_i^*|^2$ is

$$|U_d^T U_i^*|^2 = |\sin \gamma_d^* \sin \gamma_i e^{j(\eta_d^* - \eta_i)} + \cos \gamma_d^* \cos \gamma_i|^2. \quad (37)$$

Comparing this with Equation (28) and considering the steps leading up to Equation (32), we see that the SINR may now be written

$$\text{SINR} = \xi_d \left[\frac{1 + \xi_i \sin^2 \left(\frac{M_d^* M_i}{2} \right)}{1 + \xi_i} \right] \quad (38)$$

where M_d^* is the point on the Poincare sphere defined by γ_d^*, η_d^* . Thus, for this interference arrival angle, the SINR depends only on the separation between M_i and M_d^* on the Poincare sphere.

We will say that polarization M_d^* is conjugate to polarization M_d . These two polarizations differ only in that the angle η_d^* is 180° away from η_d , as seen in Equation (35). By examining Equation (9), we find that the polarization conjugate to α_d, β_d will have ellipticity and orientation given by

$$\alpha_d^* = -\alpha_d$$

$$\beta_d^* = 180^\circ - \beta_d \quad (39)$$

Thus, in Figure 8, where $\alpha_d = 15^\circ$ and $\beta_d = 30^\circ$, the conjugate polarization is $\alpha_d^* = -15^\circ$ and $\beta_d^* = 150^\circ$, and the SINR is poor when $\alpha_i \approx \alpha_d^*$ and $\beta_i \approx \beta_d^*$.

The physical explanation for this result is simple. When the interference arrives from the opposite direction to the desired signal and has conjugate polarization, it produces element signals with exactly the same relative amplitudes and phases as does the desired signal. Thus, this situation is electrically equivalent to the case where the interference arrives from the same direction and has the same polarization as the desired signal.

Special Case 3: Linear Desired Signal Polarization

The case where the desired signal is linearly polarized is the worst situation for this array. In this case, the array is vulnerable to similarly polarized interference from a wide range of angles. Moreover, it does not matter what direction the desired signal arrives from, or in what direction its (linear) polarization is oriented. The more closely the desired signal polarization approaches linear, the wider is the range of angles from which an interference signal can effectively reduce the SINR.

Let us first illustrate this with a simple example. Suppose $\theta_d = \phi_d = 90^\circ$, and $\theta_i = 90^\circ$. Also, suppose both signals are linearly polarized, $\alpha_d = \alpha_i = 0^\circ$, and the orientation angles of both signals are the same, $\beta_d = \beta_i$. Under these conditions, Figure 9 shows the SINR versus ϕ_i for several values of $\beta_d (= \beta_i)$ approaching 90° (polarization parallel to the z-oriented dipole), for SNR = 0 dB and INR = 40 dB. As may be seen in the figure, the closer β_d approaches 90° , the wider an angular separation ($\phi_i - \phi_d$) is required between the two signals to achieve a given SINR.

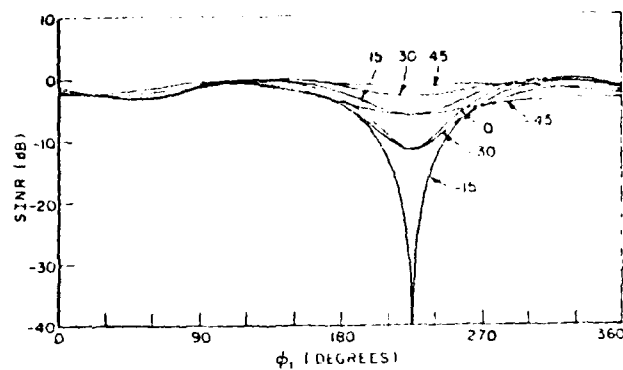


Figure 8. SINR vs. ϕ_i .

$\theta_d=45^\circ$, $\phi_d=45^\circ$, $\alpha_d=15^\circ$, $\beta_d=30^\circ$, SNR=0 dB.
 $\theta_i=135^\circ$, $\beta_i=150^\circ$, INR=40 dB.

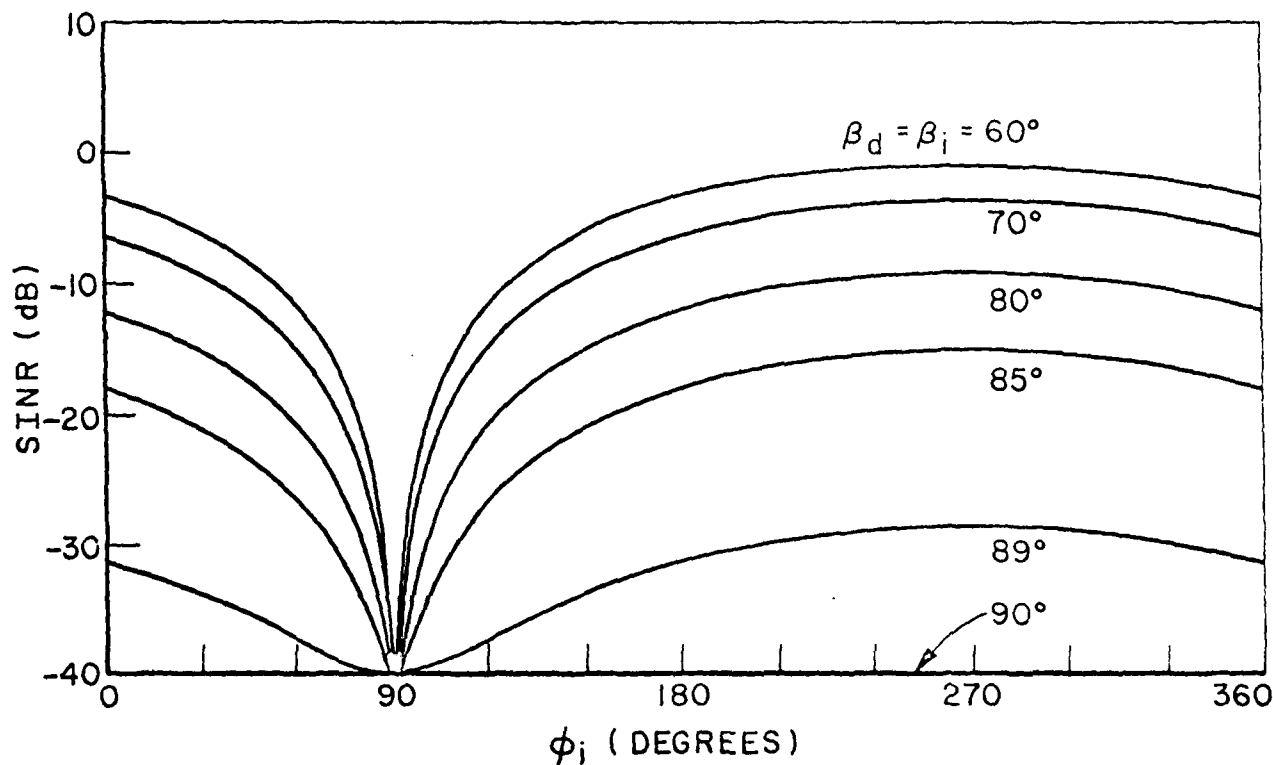


Figure 9. SINR vs. ϕ_i for $\beta_d=\beta_i$.

$\theta_d=90^\circ$, $\phi_d=90^\circ$, $\alpha_d=0^\circ$, SNR=0 dB.
 $\theta_i=90^\circ$, $\alpha_i=0^\circ$, INR=40 dB.

If the desired signal is linearly polarized but the interference arrives from an arbitrary direction with arbitrary polarization, then the SINR depends on the interference parameters in a simple way. Suppose, for example, $\theta_d = 90^\circ$, $\alpha_d = 0^\circ$, and $\beta_d = 90^\circ$. (In this case, the desired signal excites only the z-axis dipole.) Then, from Equation (9), we have $\gamma_d = 90^\circ$ and Equation (12b) yields

$$\left| U_d^T U_i^* \right|^2 = \sin^2 \theta_i \sin^2 \gamma_i \quad (40)$$

so the SINR in Equation (26) becomes

$$\text{SINR} = \xi_d \left[1 - \frac{\sin^2 \theta_i \sin^2 \gamma_i}{\xi_i^{-1} + 1} \right] \quad (41)$$

Note that this result holds regardless of ϕ_i or η_i .

Equation (41) shows that, for a given θ_i , a constant SINR will be obtained for all polarizations M_i with the same γ_i on the Poincare sphere. As may be seen from Figure 3, a locus of constant γ_i is a circle on the sphere. For $\gamma_i = 45^\circ$, for example, it is a great circle passing through the equator at $\alpha_i = 0^\circ$, $\beta_i = 45^\circ$ and 135° , and through the top and bottom poles corresponding to circular polarization. For $\gamma_i = 0^\circ$ or $\gamma_i = 90^\circ$, the circle reduces to a point, corresponding to horizontal or vertical polarization, respectively. For any given γ_i , associated values of α_i and β_i can be found from Equation (9a).

The physical reason the SINR in Equation (41) is invariant with η_i is as follows. For a given θ_i , varying the polarization parameters α_i , β_i in such a way that γ_i remains constant holds the amplitudes of the vertical and horizontal components of the incident field constant. Only the relative phase between E_θ and E_ϕ changes. For example, if $\gamma_i = 45^\circ$, the polarization

ellipse stays inside of and tangent to a square, as shown in Figure 10. The vertical component E_θ appears in $\hat{x}_3(t)$, and the horizontal component E_ϕ appears in the combined outputs from $\hat{x}_1(t)$ and $\hat{x}_2(t)$. It can be shown that, with no desired signal component in $\hat{x}_1(t)$ or $\hat{x}_2(t)$, the array combines $\hat{x}_1(t)$ and $\hat{x}_2(t)$ with maximal ratio combiner weights⁸ to yield the E_ϕ -component of the interference at maximum interference-to-noise ratio, regardless of ϕ_i . This combined output from $\hat{x}_1(t)$ and $\hat{x}_2(t)$ is adjusted to the proper phase by the weights and then subtracted from $\hat{x}_3(t)$ to null the interference. Since the amplitudes of the vertical and horizontal components are fixed as η_i varies, so is the output SINR.

Since the SINR in Equation (41) does not depend on ϕ_i , the array will be equally vulnerable to interference from any ϕ_i ; separating the two signals in ϕ does not help. The worst case occurs if $\theta_i = 90^\circ$, $\alpha_i = 0^\circ$, and $\beta_i = 90^\circ$ (so $\gamma_i = 90^\circ$), when Equation (41) gives

$$\text{SINR} = \frac{\xi_d}{\xi_i + 1} \quad (42)$$

which is essentially -40 dB if SNR = 0 dB and INR = 40 dB, again regardless of ϕ_i .

In Equation (41), we assumed that $\theta_d = 90^\circ$, $\alpha_d = 0^\circ$, and $\beta_d = 90^\circ$ but that the interference parameters are arbitrary. Alternatively, we may assume the interference is linearly polarized, say $\theta_i = 90^\circ$, $\alpha_i = 0^\circ$ and $\beta_i = 90^\circ$, and the desired signal is arbitrary. In this case, $\gamma_i = 90^\circ$ and Equation (12b) yields

$$\left| U_d^T U_i^* \right|^2 = \sin^2 \theta_d \sin^2 \gamma_d \quad (43)$$

so

$$\text{SINR} = \xi_d \left[1 - \frac{\sin^2 \theta_d \sin^2 \gamma_d}{\xi_i^{-1} + 1} \right] \quad (44)$$

regardless of ϕ_d or η_d . This result is analogous to Equation (41). With the interference linearly polarized parallel to the z-oriented dipole, it tells us how close $\sin\theta_d \sin\gamma_d$ can approach unity if a given SINR must be obtained. For example, if an SINR of -3 dB is necessary (with SNR = 0 dB and INR = 40 dB), either θ_d must be less than 45° if vertical linear polarization ($\gamma_d = 90^\circ$) is used, or, if the polarization is circular ($\gamma_d = 45^\circ$), the signal may arrive from $\theta_d = 90^\circ$.

In this example it is easy to see the reason for the poor performance of the array. When $\theta_d = 90^\circ$, $\alpha_d = 0^\circ$ and $\beta_d = 90^\circ$ (and ϕ_d has any value), the desired signal excites only the z-axis dipole. Clearly, an interference signal from $\theta_i = 90^\circ$ with the same polarization ($\alpha_i = 0^\circ$, $\beta_i = 90^\circ$) will also produce a signal only in this dipole, regardless of ϕ_i . The array will then have no ability to null one signal and not the other, whether $\phi_i = \phi_d$ or not.

However, it turns out that the poor performance of the array with a linearly polarized desired signal does not depend on having the electric field aligned with one of the dipoles*. A similar result occurs whenever the desired signal is linearly polarized, regardless of its arrival angles θ_d , ϕ_d , or orientation angle β_d . In general, with a linearly polarized desired signal, the array will be vulnerable to any linearly polarized interference signal whose electric field is parallel to that of the desired signal. More specifically, suppose the desired signal arrives from a given direction with a given linear polarization. Imagine a plane passing through the center of the tri-pole and oriented perpendicular to the desired signal electric field. Then a linearly polarized interference signal incident on the tri-pole from any direction in this plane, with its electric field perpendicular to this plane,

*The author is grateful to Andrew Zeger of Zeger and Abrams, Inc., who first pointed this out to him.

will produce a low SINR from the array. The physical reason is that such an interference signal produces the same element voltages in the array as the desired signal (except for a scale constant). Hence, a set of array weights that nulls the interference also nulls the desired signal.

Simple SINR formulas for the general case of a linearly polarized desired signal may be obtained by defining a new coordinate system whose axes are chosen to align with the desired signal. We shall not carry out the details here, which are tedious and appear to give little additional insight into the problem, but shall merely describe the procedure.

Assume the desired signal arrival direction θ_d, ϕ_d and orientation angle β_d are given. We define a new $x'y'z'$ coordinate system oriented so the x' -axis points in the direction θ_d, ϕ_d and whose z' -axis is parallel to the desired signal electric field. This coordinate system may be obtained by a sequence of three orthogonal coordinate rotations of the original xyz coordinate system in Figure 10 about each of its axes. (The three angles of rotation are usually called Eulerian angles¹².) Using the $x'y'z'$ axes, we define polar coordinates r', θ', ϕ' in the usual way, with θ' measured from the z' axis and ϕ' from the x' axis. In this coordinate system, the desired signal arrives from $\theta_d' = 90^\circ, \phi_d' = 0^\circ$ and has orientation angle $\beta_d' = 90^\circ$. The interference parameters $\theta_i', \phi_i', \gamma_i'$ and η_i' in the primed frame may be derived from the corresponding parameters in the unprimed frame by means of the Eulerian angle rotations. (Note that angle α_i , which describes the ellipticity of the interference, is the same in either frame.) To evaluate the SINR in Equation (26), we note that U_d and U_i are vectors, which may be represented in terms of either their xyz components or their $x'y'z'$ components. Since the $x'y'z'$ system is obtained from the xyz system by an orthogonal transformation, $\left| U_d^T U_i^* \right|^2$ is invariant under this transformation and may be computed

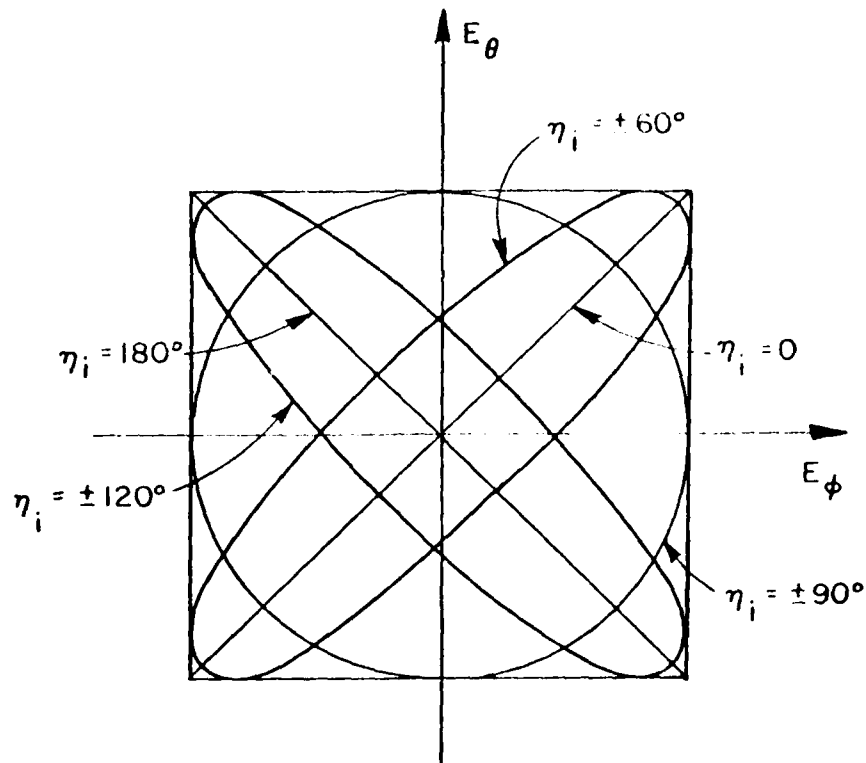


Figure 10. Polarization ellipses with $\gamma_i = 45^\circ$.

in either system with the same result. In the primed system, however, since $\theta'_d = 90^\circ$, $\alpha'_d = 0^\circ$ and $\beta'_d = 90^\circ$, the steps required to evaluate $\left| U_d^T U_i^* \right|^2$ are identical to those used to obtain Equation (40), except that all quantities are now primed. Thus, we find

$$\left| U_d^T U_i^* \right|^2 = \sin^2 \theta'_i \sin^2 \gamma'_i \quad (45)$$

and

$$\text{SINR} = \xi_d \left[1 - \frac{\sin^2 \theta'_i \sin^2 \gamma'_i}{\xi_i^{-1} + 1} \right] \quad (46)$$

where θ'_i is the polar angle and γ'_i the polarization parameter of the interference, both as seen in the primed system. Since $\theta' = 90^\circ$ is the plane perpendicular to the desired signal electric field, we see from Equation (46) that any linearly polarized interference signal arriving in this plane and polarized perpendicular to the plane (so $\beta'_i = 90^\circ$ and hence $\gamma'_i = 90^\circ$) will produce a low SINR from the array, as discussed above.

Thus, in conclusion, Special Cases 1, 2 and 3 describe the situations in which this array will not yield good performance. Other than in these cases, however, performance such as that shown in Figures 4, 5, and 8 is typical of what is obtained. In general, this array has quite a good ability to protect a desired signal from interference.

As a final remark, we note that the tripole antenna may itself be used as a building block in a larger adaptive array. For example, the poor performance of the tripole with linearly polarized signals can be eliminated by arraying two or more tripoles. Such an array will not have the difficulties described in Special Cases 2 or 3. (However, it will still have the behavior described in Special Case 1.)

IV. CONCLUSIONS

In this report we have studied the performance of the tripole antenna, an adaptive array of three mutually perpendicular dipoles. The array output SINR (signal-to-interference-plus-noise ratio) has been computed when a desired signal and an interference signal, each with arbitrary elliptical polarization, are incident on the array from arbitrary directions. Uncorrelated thermal noise is also assumed present in each element signal.

This simple array has been shown to have an impressive ability to protect a desired signal from interference. Figures 4 and 5 show typical curves of output SINR obtained with this array. The special cases in which the array output SINR is not good have also been examined. It was shown that the performance is poor in three situations:

(1) When both signals arrive from the same direction. In this case the output SINR depends only on the separation of the polarizations of the two signals on the Poincare sphere.

(2) When the two signals arrive from opposite directions. In this case the output SINR depends only on the Poincare sphere separation between the interference polarization and a polarization conjugate to that of the desired signal.

(3) When the desired signal is linearly polarized. In this case the array is vulnerable to interference from a wide range of angles.

In general, if linearly polarized desired signals are avoided, the tripole antenna will protect a desired signal from almost any interference signal. If the interference arrives from a different direction than the desired signal, the array will suppress it regardless of its polarization.

If the interference arrives from the same direction as (or opposite to) the desired signal, the array will suppress it unless its polarization is the same as (conjugate to) that of the desired signal.

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